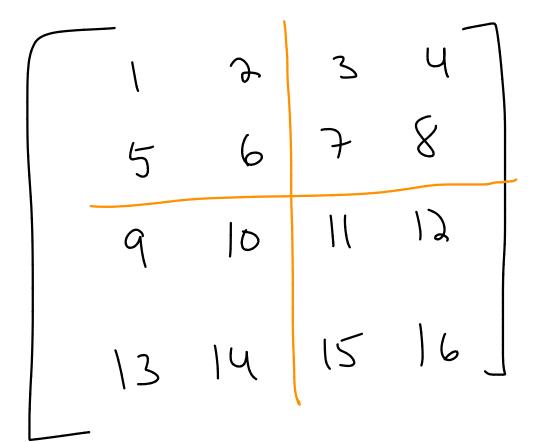
Announcements

1) HW 2 due Thursday

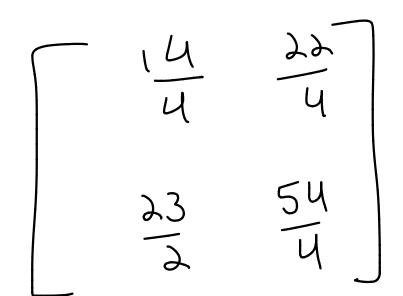
Example from last time: image compression

"large" file represented as a matrix



average over 2×2 blocks to get a smaller file

We get



Want: express this process using matrix multiplication

Matrix Multiplication The rules: Given an mxn matrix A (mrows, n columns), We can multiply A on the 1) left by a kxm matrix (same number of columns as A has rows) to get a kxn Matrix 2) right by a nxl matrix (same number of rows as A has columns) to get an mxl matrix

The idea : dot products

$$E \times unple I: A = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 8 \\ -1 & 15 \\ -6 & 4 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 3 & 8 \\ -1 & 15 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3(-1) + 8 \cdot 2 & 3 \cdot 3 + 8 \cdot 5 \\ (-1)(-1) + 15 \cdot 2 & (-1) \cdot 3 + 15 \cdot 5 \\ (-1)(-1) + 15 \cdot 2 & (-1) \cdot 3 + 15 \cdot 5 \\ (-1)(-1) + 4 \cdot 2 & (-1) \cdot (-1) + 4 \cdot 5 \end{bmatrix}$$

 $= \begin{bmatrix} 13 & 49 \\ 31 & 42 \\ 14 & 2 \end{bmatrix}$ a 3x2 natrix

In general:

IF A is man and B is kam,

$$(BA)_{i,j} = entry in the ith row, jth where of BA (14144, 14j4n)$$

Buck to compression: We started with a 4×4 matrix A undended with a 2x2 matrix D. Since we changed both the rows and columns of A (in terms of their number), We need to find matrices B and C with BAC = D $B = a \times 4$ $C = U \times 2$

Stepl: Find B, make it average-almost!
B

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 5 & 7 & 8 \\ 0 & 4 & 4 \end{bmatrix}$$

 $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 5 & 7 & 8 \\ a & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

$$-\begin{bmatrix} 6 & 8 & 10 & 13 \\ -9 & 4 & 4 & 4 \\ -23 & 34 & 36 & 28 \\ -34 & 4 & 4 & 4 \end{bmatrix}$$

Step 2: Find C, make it add
the numbers in the
blue 1x2 blocks C

$$\begin{array}{c|c}
 & & & & 10 & 10 \\
\hline
 & & & & 10 & 10 \\
\hline
 & & & & & 4 \\
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$$\begin{bmatrix} + 8 & | & | \\ 4 & | & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\ 4 & | \\$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = A$$

Want B, C where BAG = D Stepl: Find B, make it "average" $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \\ 7 & 8 & 9 \end{bmatrix}$ $= \begin{bmatrix} 5 & 1 & 1 & 1 \\ 5 & 1 & 1 & 1 \\ 7 & 8 & 9 \end{bmatrix}$

Step 2: Find C, make it "wreet" any errors from B

 $= \begin{bmatrix} 12 & 9 \\ 14 & 9 \\ 15 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 15 & 9 \\ 15 & 9 \end{bmatrix}$

Easy Path

Vectorization: take an nxm Matrix, turn it into an

NMX1 vector.

$$A = \begin{bmatrix} 4 & 3 \end{bmatrix}$$

$$A \times 2 \text{ matrix} = 4 \text{ entries}$$

$$Vector \text{ should be } 4 \times 1$$

$$V = \begin{bmatrix} -6 \\ 55 \\ 4 \\ 3 \end{bmatrix}$$

remembers all the entries of A
The position of the numbers in A
is an biguous just given V.

$$A = \begin{bmatrix} -6 & 55 \\ 4 & 3 \end{bmatrix}$$

Example 3

How do we do this using
matrices?
Want a Ux1 vector from example.
(Ux2)
$$A(2x1)$$

 $A \begin{bmatrix} 1\\ 0 \end{bmatrix} = \begin{bmatrix} -6 & 55\\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1\\ 0 \end{bmatrix}$
 $= \begin{bmatrix} -6\\ 4 \end{bmatrix}$
 $A \begin{bmatrix} 0\\ 1 \end{bmatrix} = \begin{bmatrix} 55\\ 3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} A \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 55 \\ 3 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 55 \\ 3 \end{bmatrix}$$
add the outputs to get
$$\begin{bmatrix} -4 \\ 55 \\ 3 \\ 3 \end{bmatrix}$$

This procedure vectorites A, but we don't get the vector we wanted - the order is off. To fix the order, multiply by a permutation matrix, in this case $\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 3
\end{bmatrix}$ 55 4 2

If we wanted to average the entries of A, we could multiply our vector by $\begin{bmatrix} 1 \times 4 \\ -6 \end{bmatrix} \\ \begin{bmatrix} -6 \\ 55 \\ 4 \\ 4 \end{bmatrix} \\ \begin{bmatrix} -6 \\ 55 \\ 4 \end{bmatrix} \\ 3 \end{bmatrix}$ $=\frac{1}{4}(-6+55+4+3)$ = the average !

We could write this process as
(forsetting the permutation matrix)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \land \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \land \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \land \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \land \begin{bmatrix} 0 \\ 0 \end{bmatrix} \land \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \land \begin{bmatrix} 0 \\ 0 \end{bmatrix} \land \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \land \begin{bmatrix} 0 \\ 0 \end{bmatrix} \land$$

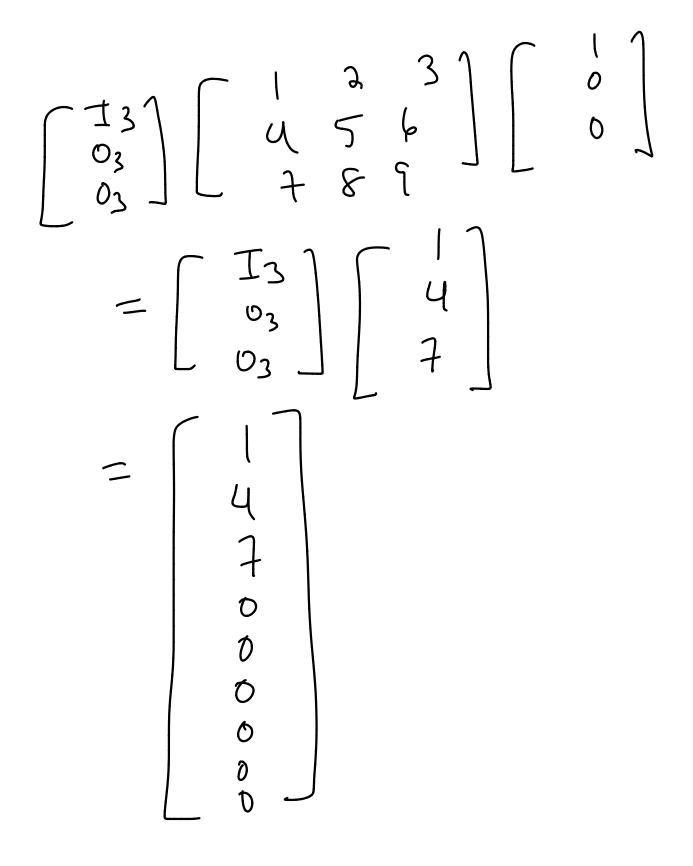
Bacy to example 2:

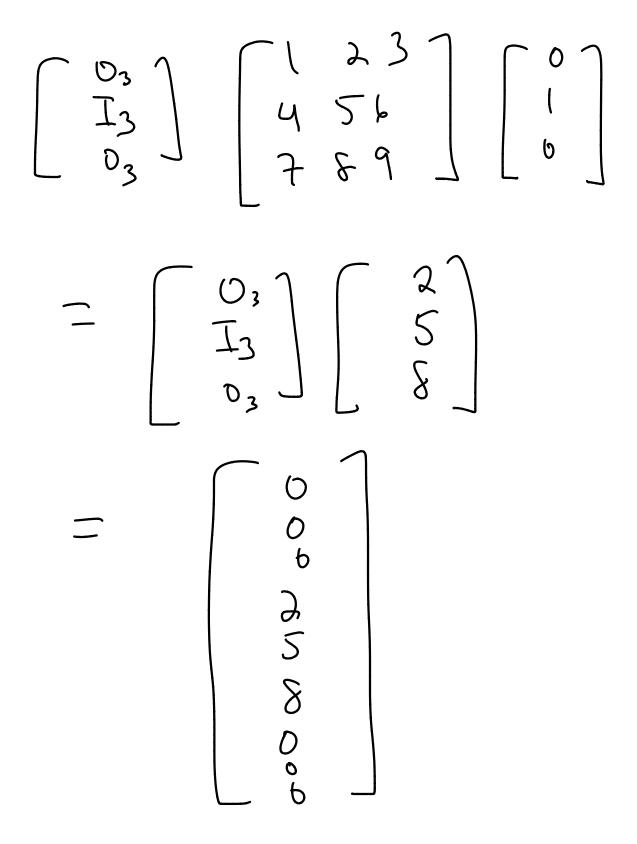
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & b \\ 7 & 8 & 9 \end{bmatrix} = A$$

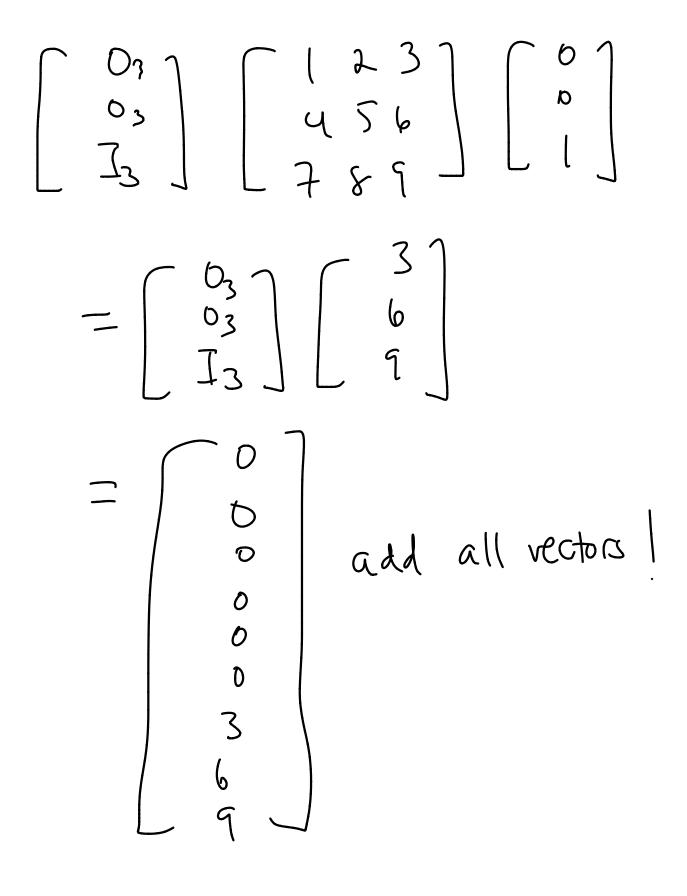
Step 1: vectorize A

$$a \times 3$$

 $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 0 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 10 \\ 0 & 0 & 1 \end{bmatrix}$
where $I_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 10 \\ 0 & 0 & 1 \end{bmatrix}$ (identity
matrix)
and $O_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (zero
matrix)







$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 \\ 9 \end{bmatrix} = A$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = A$$
A veraging gets a dxd matrix.
We need to multiply v by
(a x 9) v ((x d))
gx1

We get
$$\begin{bmatrix} 1\\ 4\\ 7\\ 2\end{bmatrix} = N$$

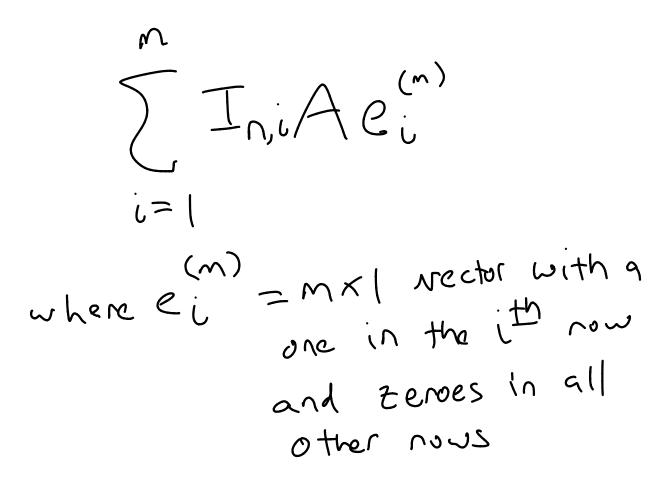
UX9 12002000 0000 よう0 2 5 Ο \bigcirc 0 0 0 0 0 0 D 8 Ь

Chent - make q Uxl vector that contains the averaged entries A. Each row should average a particular "block" of A. oF

(Seneral Implementation

To average and nxm matrix into KXR "blocks", first

vectorize.



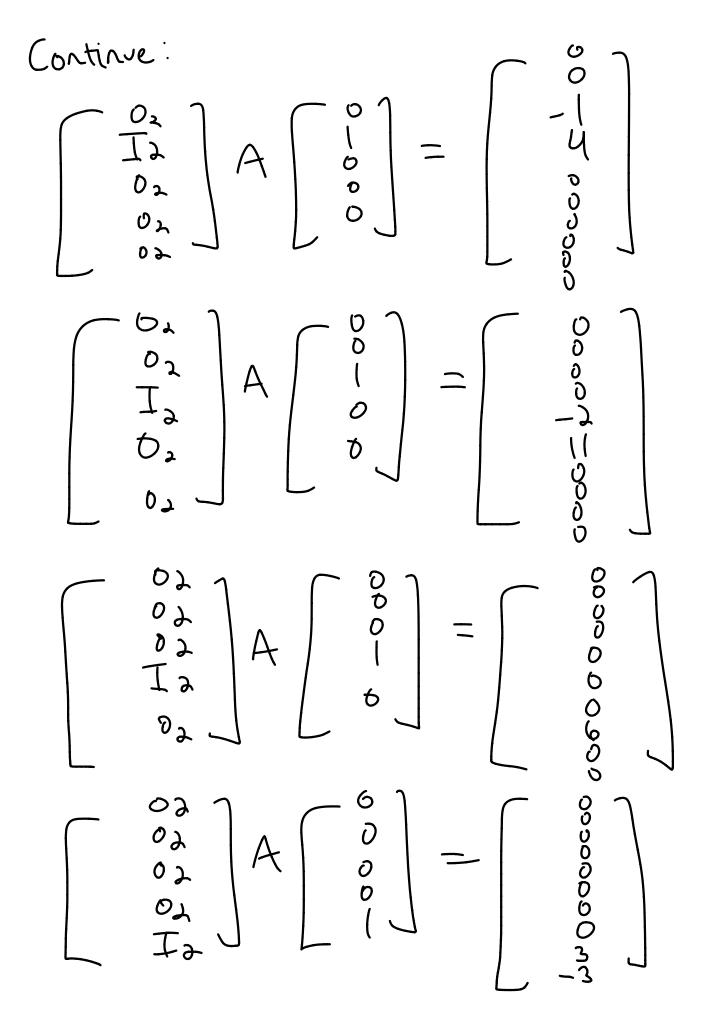
Example 4:

 $A = \begin{bmatrix} 5 & -1 & -2 & 0 & 3 \\ 8 & 4 & 11 & 6 & -3 \end{bmatrix}$

make A into a 1×3 matrix by averaging in the indicated blocks.

Express this procedure using Matrices.

Step I: vectorize A need a 10×1 vector (10×2) A (5×1) $\begin{bmatrix} J_{\lambda} \\ O_{\lambda} \\ O_$ $\begin{bmatrix} 1_{2} \\ 0_{2} \\ 0_{3} \\ 0$ 0000000000



Add all vectors to get 2 average 8 vectorited ng C 2

Step 2. Average to get a 3x1 vector. Express using

matrices.

3×1 58-(3× 10) ر ۲ 11 b 3 ζ 0 132 N N/2

Step 3 Undo vectorization - if you want!